Ghost condensation in nonlinear gauges: Euclidean space, Minkowski space, and high temperature

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(Received 12 November 2002; published 5 February 2003)

$SU(2)$ gauge theory with nonlinear gauge fixing is investigated. Since this system contains a quartic ghost interaction, there is the possibility of ghost condensation. It is shown that this condensation occurs in Euclidean space, and gauge fields acquire tachyonic masses in the presence of a ghost condensate. We also show that these tachyonic masses are gauge dependent. Contrastingly, there is no condensation in Minkowski space. However, even if we are in Minkowski space, this condensation appears at high temperature, and gauge fields get tachyonic masses.

DOI: 10.1103/PhysRevD.67.045002 PACS number(s): 11.10.Wx, 11.15.Ex

I. INTRODUCTION

In the study of non-Abelian gauge theories, the Abelian gauge is often used to investigate confinement [1]. In this gauge, renormalizability requires a quartic ghost interaction [2]. Several authors studied gauge theories in this gauge, and pointed out that the quartic ghost interaction gives rise to ghost condensation [3,4]. Self-energies of off-diagonal gluons were also calculated under the condensation [3,4]. It is concluded that these gluons acquire a mass, which implies the so-called Abelian dominance. Furthermore, in the $SU(2)$ case, Schaden discussed the relation of the condensation and the spontaneous breakdown of $SL(2,R)$ symmetry [3]. In the case of $SU(N)$, the extension of this $SL(2,R)$ symmetry was accomplished by the present author [5].

Is the ghost condensation peculiar to the Abelian gauge? It is worth studying this phenomenon in other gauges with a quartic ghost interaction. The $OSp(4|2)$ invariant gauge [6] was investigated by Kondo [7]. It was found that the ghost condensation occurs, but the Becchi-Rouet-Stora (BRS) symmetry breaks spontaneously.

In this paper, we consider nonlinear gauges [8]. Especially, we choose the gauge of Ref. [9], which includes the $OSp(4|2)$ invariant gauge as a special case. Since the BRS symmetry plays an important role in constructing renormalized gauge theories, it should be preserved. In the next section, we present a Lagrangian density which preserves the BRS symmetry even if the ghost condensation happens. Based on a one-loop effective potential, it is shown that the condensation occurs. In Sec. III, one-loop self-energies for gauge fields are calculated in the presence of a ghost condensate. It is found that gauge fields exhibit tachyonic masses in the infrared region. We also find that the mass found in Ref. [3] is tachyonic. These results are obtained in Euclidean space. The Minkowski case is studied in Sec. IV. If we carry out a straightforward calculation, the Minkowski metric prevents the ghost condensation. Despite the negative result of Sec. IV, this condensation may happen in the real world at high temperature. In Sec. V, we show that the ghost condensation occurs at high temperature. As in the Euclidean case, gauge fields have tachyonic masses in the infrared region. Section VI is devoted to summary and comments. In Appendix A, additional explanations of the Lagrangian studied in this paper are given. From Appendix B to Appendix E, calculational details are presented.

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II. NONLINEAR GAUGES AND GHOST CONDENSATION IN EUCLIDEAN SPACE

We study a non-Abelian gauge theory in Euclidean space. A Lagrangian density is

$$\mathcal{L}=\mathcal{L}_{\rm inv}+\mathcal{L}_{gfgh}, \quad \mathcal{L}_{\rm inv}=\frac{1}{4}F_{\mu\nu}\cdot F_{\mu\nu}.$$

The gauge-fixing and ghost part is

$$\mathcal{L}_{gfgh}=-i\,\delta_B(\bar{c}\cdot F), \tag{1}$$

where F is a gauge-fixing function. We focus on the $SU(2)$ case, and use the following notation:

$$F\cdot G=F^A G^A, \quad (F\times G)^A=f^{ABC}F^B G^C, \quad A=1,2,3.$$

Here the summation with respect to repeated indices is implied. The BRS and the anti-BRS transformations [10] are

$$\delta_B A_\mu=D_\mu c, \quad \delta_B c=-\frac{g}{2}c\times c,$$

$$\delta_B\bar{c}=iB, \quad \delta_B\bar{B}=g\bar{B}\times c, \tag{2}$$

$$\bar{\delta}_B A_\mu=D_\mu\bar{c}, \quad \bar{\delta}_B\bar{c}=-\frac{g}{2}\bar{c}\times\bar{c},$$

$$\bar{\delta}_B c=i\bar{B}, \quad \bar{\delta}_B B=gB\times\bar{c}, \tag{3}$$

where $\bar{B}=-B+ig\bar{c}\times c$. To obtain a covariantly gauge-fixed action, F is often chosen as $\partial_\mu A_\mu-(\alpha/2)B$. However, taking account of the correspondence between the transformations δ_B and $\bar{\delta}_B$, it is natural to include $\bar{B}$ into a gauge-fixing function as [5]

$$F=\partial_\mu A_\mu-\frac{\alpha_1}{2}B+\frac{\alpha_2}{2}\bar{B}-w, \tag{4}$$

where α_1 and α_2 are gauge parameters, and w is a constant determined below [11]. From Eqs. (1) and (4), we obtain

$$\mathcal{L}_{gfgh}=B\cdot\partial_\mu A_\mu+i\overline{c}\cdot\partial_\mu D_\mu c-\frac{\alpha_1}{2}B^2-\frac{\alpha_2}{2}\overline{B}^2-Bw. \quad (5)$$

If we set $w=0$, Eq. (5) becomes the Lagrangian discussed in Ref. [9]. Furthermore, the choice $\alpha_1=\alpha_2=\alpha/2$ leads to the gauge-fixing term which is based on the $OSp(4|2)$ spacetime supersymmetry [6].

As $\overline{B}^2$ in Eq. (5) contains a quartic ghost interaction, the ghost condensation may happen. To see it, the auxiliary field φ is introduced as

$$\mathcal{L}_{gfgh}=B\cdot(\partial_\mu A_\mu+\varphi-w)-\frac{\alpha_1}{2}B^2+i\overline{c}\cdot\partial_\mu D_\mu c -ig\varphi\cdot(c\times\overline{c})+\frac{\varphi^2}{2\alpha_2}. \quad (6)$$

It is important to use the auxiliary field φ, which becomes $\alpha_2\overline{B}$ if the equation of motion for φ is used. In Appendix A, this point is elucidated, and, to make clear the meaning of φ from another view point, Eq. (6) is derived by a functional integral method. Now we explain the importance of the constant w. In the $OSp(4|2)$ invariant gauge [6], it was shown that the BRS symmetry breaks spontaneously if $igc\times\overline{c}$ acquires a vacuum expectation value [7]. However, we can preserve the BRS symmetry manifestly in Eq. (6), even if φ acquires a vacuum expectation value. To explain it, we add the subscripts B and R to bare and renormalized quantities, respectively. If we divide φ into a classical part φ_0 and a quantum part φ' as $\varphi(x)=\varphi_0+\varphi'(x)$, we can choose w to cancel the divergent classical part [11]:

$$\varphi_{0B}-w=Z_{\varphi_0}^{1/2}\varphi_{0R}-w=0.$$

Thus $\langle 0|B_B|0\rangle\propto\langle 0|\partial_\mu A_{B\mu}+\varphi'_B|0\rangle=0$ is guaranteed, and the BRS symmetry is preserved [11].

Hereafter, we neglect the subscript R, for simplicity. Then the (anti)ghost bilinear part $i\overline{c}(\Box+g\varphi_0\times)c$ gives the effective action $-\mathrm{Tr}[\ln(\Box+g\varphi_0\times)]$ at the one-loop level. Including this contribution, a one-loop effective potential becomes

$$V_E(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}-V_{\mathrm{gh}}(\varphi_0),$$

$$V_{\mathrm{gh}}=\int\frac{d^4k}{(2\pi)^4}\ln[(-k^2)^2+g^2\varphi_0^2]. \quad (7)$$

We can calculate Eq. (7) in some ways. One way is to use Eq. (B6). Using dimensional regularization, we obtain

$$V_E(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}-\frac{(g\varphi_0)^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}+\frac{3}{2}-\gamma+\ln 4\pi-\frac{1}{2}\ln(g\varphi_0)^2\right), \quad (8)$$

where γ is the Euler number, and $\varepsilon=(4-D)/2$ with $D\to 4$. We define renormalization constants for φ_0 and α_2 as $\varphi_{0B}=Z_{\varphi_0}^{1/2}\varphi_0$ and $\alpha_{2B}=Z_{\alpha_2}\alpha_2$. Since

$$\frac{\varphi_{0B}^2}{2\alpha_{2B}}=\frac{\varphi_0^2}{2\alpha_2}-\frac{1-Z_{\varphi_0}Z_{\alpha_2}^{-1}}{2\alpha_2}\varphi_0^2$$

holds, these renormalization constants are chosen so that the counterterm

$$-\frac{1-Z_{\varphi_0}Z_{\alpha_2}^{-1}}{2\alpha_2}\varphi_0^2$$

subtracts the divergence in Eq. (8). Thus we set

$$\frac{1-Z_{\varphi_0}Z_{\alpha_2}^{-1}}{2\alpha_2}\varphi_0^2=-\frac{(g\varphi_0)^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}-\gamma+\ln 4\pi-\frac{1}{2}\ln(\kappa\mu^2)^2\right), \quad (9)$$

where μ is a renormalization scale. Subtracting Eq. (9) from Eq. (8), the renormalized effective potential

$$V_E(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}+\frac{(g\varphi_0)^2}{32\pi^2}\left[\ln\left(\frac{g\varphi_0}{\kappa\mu^2}\right)^2-3\right], \quad (10)$$

which satisfies the renormalization condition $V''(\varphi_0=\kappa\mu^2/g)=1/\alpha_2$, is obtained. The condition $V_E'(\varphi_0)=0$ gives $|g\varphi_0|=0$ and

$$|g\varphi_0|=\kappa\mu^2 e^{1-8\pi^2/\alpha_2 g^2}. \quad (11)$$

From $V_E''(\varphi_0)$, we find that the latter gives the minimum of $V_E(\varphi_0)$. Hence, φ condenses.

The choice $\kappa=4\pi e^{-\gamma}$ corresponds to the minimal subtraction scheme. Using this scheme, and choosing the condensate in the third direction of $SU(2)$, Eq. (11) gives

$$(g\varphi_0^A)=(0,0,v),\quad v=4\pi\mu^2 e^{1-\gamma}e^{-8\pi^2/\alpha_2 g^2}. \quad (12)$$

This condensation has the same form obtained in Refs. [3], [4] (maximal Abelian gauge) and [7] [$OSp(4|2)$ invariant gauge]. We use the choice (12) henceforth.

We note that the condition $V'(\varphi_0)=0$ gives a gap equation, which becomes

$$\frac{v}{g\alpha_2}-\int\frac{d^4k}{(2\pi)^4}\frac{2gv}{(-k^2)^2+v^2}=0. \quad (13)$$

Although the auxiliary field φ is different from a usually used one [3,4,7], the effective potential (8) has the same structure that was obtained in these references. Therefore similar relations between anomalous dimensions found in Ref. [4] hold. First, we consider $\gamma_{\alpha_2}=\mu(\partial/\partial\mu)\alpha_2$ and $\gamma_{\varphi_0}=\frac{1}{2}\mu(\partial/\partial\mu)\ln Z_{\varphi_0}$. From Eq. (9), we find

$$\frac{1-Z_{\varphi_0}Z_{\alpha_2}^{-1}}{2\alpha_2}=-\frac{g^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}-\gamma+\ln 4\pi-\frac{1}{2}\ln(\kappa\mu^2)^2\right).$$

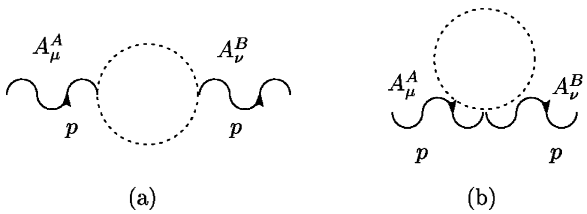

FIG. 1. The diagrams which contribute to the one-loop self-energy of gauge fields: (a) in the nonlinear gauge, and (b) in the maximal Abelian gauge.

If the operator $\mu(\partial/\partial\mu)$ operates on this expression, it gives the relation

$$\gamma_{\alpha_2}+2\alpha_2\gamma_{\varphi_0}=-\frac{(\alpha_2 g)^2}{4\pi^2}. \tag{14}$$

Next, we consider the anomalous dimension of v which is defined by $\mu(\partial/\partial\mu)v=-\gamma_v v$. Since $v=g\varphi_0$, the left hand side is rewritten as $[\beta(g)(\partial/\partial g)-\gamma_{\varphi_0}\varphi_0(\partial/\partial\varphi_0)]v$. Thus we obtain

$$\gamma_v=-\frac{\beta(g)}{g}+\gamma_{\varphi_0}. \tag{15}$$

If we require $\gamma_v=0$, we find that Eqs. (14) and (15) lead to $\mu[\partial V(\varphi_0)/\partial\mu]=0$, and

$$\gamma_{\alpha_2}=\frac{\alpha_2 g^2}{4\pi^2}\left(\frac{\beta_0}{2}-\alpha_2\right), \tag{16}$$

where $\beta(g)=-\beta_0 g^3/(16\pi^2)$ has been used [3,4].

III. SELF-ENERGIES OF GAUGE FIELDS

A. One-loop calculation

In this section, we calculate the self-energies of gauge fields in the presence of the ghost condensate. In the maximal Abelian gauge, the diagram of Fig. 1(b) contributes at the one-loop level [3,4]. However, in the present gauge, the vertex which contributes to the self-energies is the usual gauge-ghost-antighost three-point vertex. Therefore the diagram to be calculated is the one depicted in Fig. 1(a).

Under the condensate $v\neq 0$, ghost propagators are

$$-iG^{AB}(k)=-i\begin{pmatrix}\frac{-k^2}{(-k^2)^2+v^2} & \frac{v}{(-k^2)^2+v^2} & 0\\ \frac{-v}{(-k^2)^2+v^2} & \frac{-k^2}{(-k^2)^2+v^2} & 0\\ 0 & 0 & \frac{1}{-k^2}\end{pmatrix}. \tag{17}$$

Using Eq. (17), the diagram in Fig. 1(a) gives the self-energy

$$-\Pi^{AB}_{\mu\nu}(p)=-\int\frac{d^4k}{(2\pi)^4}gf^{ACD}k_\mu gf^{BEF}(k-p)_\nu \times(-i)G^{CF}(k)(-i)G^{ED}(k-p). \tag{18}$$

First, as $G^{a3}(k)=G^{3a}(k)=0$ $(a=1,2)$ and $G^{12}(k)=-G^{21}(k)$, the results $\Pi^{a3}_{\mu\nu}=0$ $(a=1,2)$ and $\Pi^{12}_{\mu\nu}(p)=0$ are easily understood. Next, to compare with a self-energy in the maximal Abelian gauge [3,4], we study the components

$$-\Pi^{11}_{\mu\nu}(p)=-\Pi^{22}_{\mu\nu}(p) =-g^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(-k^2)^2+v^2} \times\frac{2k_\mu k_\nu-k_\mu p_\nu-p_\mu k_\nu}{(k-p)^2}. \tag{19}$$

The calculation of $\Pi^{11}_{\mu\nu}$ is outlined in Appendix C. The result is

$$-\Pi^{11}_{\mu\nu}(p)=-\Pi_{\mu\nu}(p,0)-\Pi_{\mu\nu}(p,v) \tag{20}$$

$$-\Pi_{\mu\nu}(p,0)=-\frac{g^2}{(4\pi)^2}\left[-\frac{p^2}{6}\delta_{\mu\nu}-\frac{1}{3}\left(p_\mu p_\nu+\frac{p^2}{2}\delta_{\mu\nu}\right)\times\left(\frac{1}{\varepsilon}-\gamma+\ln 4\pi+\frac{5}{3}-\ln p^2\right)\right], \tag{21}$$

$$-\Pi_{\mu\nu}(p,v)=-\frac{g^2}{(4\pi)^2}\left[\frac{p_\mu p_\nu}{6}\left(\frac{2v}{p^2}u_1(p,v)+u_2(p,v)\right) -\frac{\delta_{\mu\nu}}{12}[vu_1(p,v)+2p^2u_2(p,v) +3vu_3(p,v)]\right], \tag{22}$$

where

$$u_1(p,v)=\frac{v}{p^2}\left[\ln\left(1+\frac{p^4}{v^2}\right)+\frac{v}{p^2}\left(\pi-2\tan^{-1}\frac{v}{p^2}\right)-2\right], \tag{23}$$

$$u_2(p,v)=\ln\left(1+\frac{v^2}{p^4}\right)+\frac{v^2}{p^4}\ln\left(1+\frac{p^4}{v^2}\right),$$

$$u_3(p,v)=2\tan^{-1}\frac{v}{p^2}-\frac{p^2}{v}\ln\left(1+\frac{v^2}{p^4}\right).$$

The $v=0$ part $-\Pi_{\mu\nu}(p,0)$ is the self-energy obtained in the generalized Lorentz gauge. Therefore, together with the contribution of gluon 1-loop diagrams, this part yields a wave function renormalization.

New effects come from the v-dependent part $-\Pi_{\mu\nu}(p,v)$. If we take the limit $p^2\to\infty$, we find

$$-\Pi_{\mu\nu}(p,v)\rightarrow\frac{g^2}{(4\pi)^2}\left(\delta_{\mu\nu}-2\frac{p_\mu p_\nu}{p^2}\right)\frac{v^2}{2p^2}\ln\frac{p^2}{v}.$$

The asymptotic behavior $\propto(v^2/2p^2)\ln(p^2/v)$, which comes from $u_1(p,v)$ and $u_2(p,v)$, is not produced by particles with usual masses [12].

In the limit $p^2\rightarrow 0$, we obtain

$$-\Pi_{\mu\nu}(p,v)\rightarrow\frac{g^2}{(4\pi)^2}\left[-p_\mu p_\nu\left(\frac{1}{2}+\frac{1}{3}\ln\frac{v}{p^2}\right)+\delta_{\mu\nu}\left(\frac{\pi v}{4}+\frac{p^2}{3}\ln\frac{v}{p^2}+\frac{p^2}{4}\right)\right].$$

Let us study the leading term

$$-\Pi_{\mu\nu}(p\rightarrow 0,v)=\frac{g^2 v}{64\pi}\delta_{\mu\nu}, \tag{24}$$

which comes from $u_3(p\rightarrow 0,v)$, in detail. In the maximal Abelian gauge, the corresponding one-loop self-energy is

$$\frac{g^2 v}{16\pi}\delta_{\mu\nu}, \tag{25}$$

which is interpreted as a squared mass of the off-diagonal gauge fields A^a_μ $(a=1,2)$ in Refs. [3,4]. However, the difference of the factor $\frac{1}{4}$ between Eqs. (24) and (25), which stems from the replacement $k_\mu k_\nu\rightarrow(k^2/4)\delta_{\mu\nu}$ in the integral (C2), implies that it is gauge dependent. Therefore it is difficult to consider it as a physical quantity. Furthermore, this self-energy must give a tachyonic mass, because

$$D+D(-\Pi)D+D(-\Pi)D(-\Pi)D+\cdots\sim\frac{1}{p^2+\Pi},$$

where $D\sim 1/p^2$ and $\Pi\sim -g^2v/(64\pi)$. Since the sign of the self-energy is crucial, we check it in the next subsection.

B. Tachyonic mass term

The (anti)ghost bilinear part $i\bar{c}(\partial_\mu D_\mu+g\varphi_0\times)c$ gives the effective action for A^A_μ as

$$-\int d^4x\int\frac{d^4k}{(2\pi)^4}\ln[(-k^2)^2+v^2+2igvk_\mu A^3_\mu-g^2k_\mu k_\nu A^A_\mu A^A_\nu].$$

The last term $-g^2k_\mu k_\nu A^A_\mu A^A_\nu$ yields the "mass" term

$$S^1_{\rm mass}=\int d^4x g^2 A^A_\mu A^A_\nu\int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(-k^2)^2+v^2}. \tag{26}$$

We can perform the k integral by using the variable $K=k^2$, or using Eqs. (B6) and (C4). The result is

$$S^1_{\rm mass}=\int d^4x\frac{1}{2}\left(-\frac{g^2v}{64\pi}\right)A^A_\mu A^A_\mu,$$

which is tachyonic. In the same way, the term $2igvk_\mu A^3_\mu$ yields the tachyonic mass term

$$S^2_{\rm mass}=\int d^4x(-2g^2v^2)A^3_\mu A^3_\nu\int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{[(-k^2)^2+v^2]^2}=\int d^4x\frac{1}{2}\left(-\frac{g^2v}{64\pi}\right)A^3_\mu A^3_\mu. \tag{27}$$

Combining $S^1_{\rm mass}$ and $S^2_{\rm mass}$, we obtain the tachyonic mass term

$$\int d^4x\frac{1}{2}(-m_A^2)(A^a_\mu A^a_\mu+2A^3_\mu A^3_\mu),\quad m_A^2=\frac{g^2v}{64\pi}, \tag{28}$$

where $A^a_\mu A^a_\mu=A^1_\mu A^1_\mu+A^2_\mu A^2_\mu$. Thus, not only the components $A^a_\mu(a=1,2)$ but also the component A^3_μ have the tachyonic masses in the present gauge.

We can apply the same method in the maximal Abelian gauge. The related term in this gauge is [4]

$$i\bar{c}^a\{[D_\mu(A)D_\mu(A)]^{ab}-\varepsilon^{ab}v-g^2\varepsilon^{ac}\varepsilon^{db}A^c_\mu A^d_\mu\}c^b,$$

where $D_\mu(A)^{ab}=\partial_\mu\delta^{ab}-g\varepsilon^{ab}A^3_\mu$. If the ghost and the antighost are integrated out, we obtain

$$e^{-S_{\rm eff}}=\det[\Box^2+v^2+g^2\Box(A^a_\nu)^2-2g^2(\partial_\mu)^2(A^3_\nu)^2+4g^2(\partial_\mu A^3_\mu)^2+4gv\,\partial_\mu A^3_\mu].$$

Therefore an effective mass term becomes

$$S_{\rm mass}=\int d^4x\left[g^2(A^a_\mu)^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(-k^2)^2+v^2}+g^2(A^3_\mu)^2\int\frac{d^4k}{(2\pi)^4}\left(\frac{-k^2}{(-k^2)^2+v^2}-\frac{2v^2k^2}{[(-k^2)^2+v^2]^2}\right)\right]=\int d^4x\frac{1}{2}(-4m_A^2)A^a_\mu A^a_\mu,$$

where m_A^2 is defined in Eq. (28). As was stated, the tachyonic mass term for A^a_μ appears in the maximal Abelian gauge, too. In contrast with Eq. (28), A^3_μ has no tachyonic mass in the maximal Abelian gauge.

What is the meaning of these tachyonic masses? A short-distance tachyonic gluon mass was introduced phenomenologically in Ref. [13]. The relation between this tachyonic mass and the condensate $\langle(A^A_\mu)^2\rangle$ was also discussed [14]. As the tachyonic masses in Eq. (28) appear in the infrared region, they might have no relation with tachyonic mass of Ref. [13]. Since a tachyonic mass indicates an instability anyway, the above result might imply that, instead of gluonic states, hadronized states should be used in the infrared region. The relation between the ghost condensation and confinement will be discussed in another paper.

IV. MINKOWSKI SPACE

In Sec. II, we have shown that the ghost condensation happens in Euclidean space. However, in Minkowski space, a different result is derived. To see it, we consider the one-loop effective potential in Minkowski space:

$$V_M(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}+iV_{\rm gh}, \tag{29}$$

where $V_{\rm gh}$ is defined in Eq. (7). Even if $k^2=k_0^2-\mathbf{k}^2$ vanishes, the integrand $\ln[(-k^2)^2+v^2]$ is regular if $v\neq 0$. Therefore, the one-loop contribution $iV_{\rm gh}$ must be imaginary. In fact, in Appendix D, we show that

$$iV_{\rm gh}=\int\frac{d^3k}{(2\pi)^3}(E_+^u+E_-^u), \tag{30}$$

where $E_\pm^s$ $(s=u,l)$ are the square roots of $(\mathbf{k}^2\pm iv)$ defined in Eq. (D4). As $E_+^u+E_-^u$ is imaginary, $iV_{\rm gh}$ is also imaginary.

Now we calculate $V_{\rm gh}$ explicitly. Although Eq. (B6) was applied in Sec. II, Eq. (B7) must be used here. Taking the limit $a\to 0$ in Eq. (B7), we find

$$\int\frac{d^Dk}{(2\pi)^D}\ln(-k^2\pm iv)=\pm i\frac{\Gamma\left(-\frac{D}{2}\right)}{(4\pi)^{D/2}}(\pm iv)^{D/2}. \tag{31}$$

Equation (31) contains divergences. However, when Eq. (31) is substituted into $iV_{\rm gh}$, these divergences cancel out. This cancellation is due to the first $\pm$ sign on the right hand side of Eq. (31), which is absent in the Euclidean formula (B6). Thus we obtain

$$iV_{\rm gh}=\int\frac{d^Dk}{(2\pi)^D}[\ln(-k^2+iv)+\ln(-k^2-iv)]$$
$$=\frac{v^2}{32\pi^2}\ln(-1). \tag{32}$$

As stated, $iV_{\rm gh}$ is imaginary. From Eqs. (29), (32) and (C4), we obtain

$$V_M(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}-i\frac{(g\varphi_0)^2}{32\pi}. \tag{33}$$

It is evident that $V_M'(\varphi_0)=0$ gives $\varphi_0=0$. That is, the ghost condensation does not happen in Minkowski space.

V. HIGH TEMPERATURE CASE

A. Ghost condensation

At high temperature, a gauge theory in Minkowski space reduces to a gauge theory with scalar fields in three-dimensional Euclidean space [15]. If temperature $T=1/\beta$ is high, a leading contribution comes from an effective three-dimensional Euclidean part of the system. Therefore, we can apply Eq. (B6) instead of Eq. (B7) to the leading part, and the ghost condensation may occur.

We use the imaginary time formalism. The effective potential is

$$V_T(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}-V_{\rm gh}^\beta, \tag{34}$$

$$V_{\rm gh}^\beta=T\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\ln[(\mathbf{k}^2+\omega_n^2)^2+(g\varphi_0)^2], \tag{35}$$

where $\omega_n=2\pi nT$ with $n=$integer, and $g\varphi_0$ is chosen as $g\varphi_0=(0,0,v)$ with $v\geqslant 0$. We notice, in contrast with Eq. (30), Eq. (35) is real.

In Appendix E, the integral

$$J_\pm(v)=T\sum_n\int\frac{d^Dk}{(2\pi)^D}\ln(\mathbf{k}^2+\omega_n^2\pm iv) \tag{36}$$

is calculated for $D\to 3$. At high T, the result is given in Eq. (E7). As $V_{\rm gh}^\beta=J_+(v)+J_-(v)$ with $D=3$, we find

$$V_{\rm gh}^\beta=-\frac{2\pi^2}{45\beta^4}-\frac{1}{6\pi\beta}[(iv)^{3/2}+(-iv)^{3/2}]$$
$$+\frac{v^2}{16\pi^2}\left(\frac{1}{\varepsilon}+\gamma-\ln 4\pi+\ln\beta^2\right)+O(\beta^2v^3). \tag{37}$$

Substituting Eq. (37) into Eq. (34), the potential becomes

$$V_T(\varphi_0)=\frac{\varphi_0^2}{2\alpha_2}+\frac{2\pi^2}{45\beta^4}-\frac{\sqrt{2}v^{3/2}}{6\pi\beta}-\frac{v^2}{16\pi^2}$$
$$\times\left(\frac{1}{\varepsilon}+\gamma-\ln 4\pi+\ln\beta^2\right)+O(\beta^2v^3). \tag{38}$$

To obtain the term of $O(v^{3/2}/\beta)$, $v^{1/2}e^{\pm i\pi/2}$ has been used as the square root $(\pm iv)^{1/2}$. This term, which is the contribution of the effective three-dimensional part, comes from the $n=0$ part in Eq. (35):

$$T\int\frac{d^3k}{(2\pi)^3}\ln[(\mathbf{k}^2)^2+v^2].$$

This integral can be calculated directly, and gives the above term. That is, the above choice of the arguments is necessary to obtain the correct result.

Since Eq. (38) contains the divergence, we rewrite it as

$$V_T(\varphi_0)=V_E(\varphi_0)+V_\beta(\varphi_0). \tag{39}$$

Here, $V_E(\varphi_0)$ is the Euclidean effective potential (8), and a finite temperature contribution is

$$V_\beta(\varphi_0)=\frac{2\pi^2}{45\beta^4}-\frac{\sqrt{2}v^{3/2}}{6\pi\beta}-\frac{v^2}{16\pi^2}\ln v\beta^2+\frac{cv^2}{16\pi^2}+O(\beta^2v^3), \tag{40}$$

where $c=3/2+2\ln 4\pi-2\gamma$ [16]. $V_E(\varphi_0)$ is renormalized as Eq. (10). Thus, at high temperature with $\sqrt{v}\beta\ll 1$, we find

$$V_T(\varphi_0)=\frac{v^2}{2g^2\alpha_2}-\frac{\sqrt{2}v^{3/2}}{6\pi\beta}+O(v^2\ln\sqrt{v}\beta)+\cdots. \tag{41}$$

The condition $V_T'(\varphi_0)=0$ gives $v=0$ and

$$v\simeq\left(\frac{\sqrt{2}\alpha_2}{4\pi}g^2T\right)^2. \tag{42}$$

At $\varphi_0=v/g$ with v given by Eq. (42), we find $V_T''(\varphi_0)=1/(2\alpha_2)+O(g^2\ln\alpha_2 g^2)>0$. Therefore, the ghost condensate (42) appears at high temperature.

Before closing this subsection, we comment on the relation between the potential at finite temperature (34) and that in Minkowski space (29). Since the formula

$$\sum_{n=-\infty}^{\infty}\frac{1}{z^2+(n\pi)^2}=\frac{\coth z}{z}$$

holds for the complex variable $z^2=(\mathbf{k}^2\pm iv)/(2T)^2$, we apply it to Eq. (E1). The result is

$$\frac{dJ_\pm(v)}{dv}=\frac{\pm i}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{E_\pm^s}\coth\left(\frac{E_\pm^s}{2T}\right),$$

where $E_\pm^s$ $(s=u,l)$ are defined in Eq. (D4). After integrating it by v, we obtain

$$J_\pm(v)=J_\pm^0+J_\pm^\beta,$$

$$J_\pm^0=\int\frac{d^Dk}{(2\pi)^D}E_\pm^s, \tag{43}$$

$$J_\pm^\beta=\frac{2}{\beta}\int\frac{d^Dk}{(2\pi)^D}\ln(1-e^{-\beta E_\pm^s}). \tag{44}$$

As usual, $J_\pm^0$ is a $T=0$ part, and $J_\pm^\beta$, which vanishes at the low temperature limit $T=1/\beta\to 0$, is a finite temperature part [16]. When $D=3$, $-(J_+^0+J_-^0)$ in $-V_{\rm gh}^\beta$ coincides with Eq. (30). Therefore, we have to choose $E_\pm^l=-E_\pm^u$ as $E_\pm^s$ in Eqs. (43) and (44). Thus we find

$$V_T(\varphi_0)=V_M(\varphi_0)-\frac{2}{\beta}\int\frac{d^3k}{(2\pi)^3}\ln(1-e^{-\beta E_+^l})(1-e^{-\beta E_-^l}).$$

B. Magnetic mass?

The mass scale obtained in Eq. (42) is $\sqrt{v}\sim O(g^2T)$. We note that $g\sqrt{T}$ is the gauge coupling constant of the effective three-dimensional theory. The mass of $O(g^2T)$ has been discussed as a magnetic mass to relieve the infrared problem of high temperature QCD [15,17]. Is the condensation (42) related to the magnetic mass? To see it, we study the self-energies of the gauge fields in the infrared limit $\mathbf{p}\to\mathbf{0}$. As is known, the infrared problem is caused by the magnetostatic components $A_i^A(\mathbf{x})$, i.e. the time-independent space components of the gauge fields $A_\mu^A(\tau,\mathbf{x})$ [15,17]. Therefore, we calculate the effective "mass" terms (26) and (27) for the magnetostatic components $A_i^A(\mathbf{x})$. In the present case, Eq. (26) gives

$$\begin{aligned}S_{\rm mass}^1&\sim\frac{1}{2}\int_0^\beta d\tau\int d^3xA_i^A(\mathbf{x})A_j^A(\mathbf{x})\\&\quad\times 2g^2T\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\frac{k_ik_j}{[(2n\pi T)^2+\mathbf{k}^2]^2+v^2}\\&=\frac{1}{2}\int_0^\beta d\tau\int d^3xA_j^A(\mathbf{x})A_j^A(\mathbf{x})\\&\quad\times\frac{2}{3}g^2T\int\frac{d^3k}{(2\pi)^3}\left(\frac{\mathbf{k}^2}{(\mathbf{k}^2)^2+v^2}\right.\\&\quad\left.+2\sum_{n=1}^{\infty}\frac{\mathbf{k}^2}{[(2n\pi T)^2+\mathbf{k}^2]^2+v^2}\right).\end{aligned}$$

At high temperature ($T\gg\sqrt{v}$), we can carry out the k integral. The result is

$$\begin{aligned}S_{\rm mass}^1&\sim\frac{1}{2}\int_0^\beta d\tau\int d^3xA_j^A(\mathbf{x})A_j^A(\mathbf{x})\\&\quad\times\left(-\frac{\sqrt{2}}{12\pi}g^2T\sqrt{v}+\frac{1}{12}g^2T^2+O[(g\beta v)^2]\right),\end{aligned} \tag{45}$$

where the first term comes from the static $(n=0)$ part, and the nonstatic $(n\neq 0)$ contribution gives the remaining terms. We note that the second term in Eq. (45) is the largest term at high T. However, as is known, the 1-loop contribution of the gauge fields cancels out this term. That is, magnetostatic gluons do not acquire a Debye mass of $O(gT)$ [17]. In the same way, Eq. (27) leads to the expression

$$S^2_{\rm mass}\sim\frac{1}{2}\int_0^\beta d\tau\int d^3x A_i^3(\mathbf{x})A_j^3(\mathbf{x})\times(-4g^2v^2T)\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\times\frac{k_ik_j}{\{[(2n\pi T)^2+\mathbf{k}^2]^2+v^2\}^2}=\frac{1}{2}\int_0^\beta d\tau\int d^3x A_j^3(\mathbf{x})A_j^3(\mathbf{x})\times\left(-\frac{\sqrt{2}}{24\pi}g^2T\sqrt{v}+O[(g\beta v)^2]\right),\tag{46}$$

where the first term comes from the $n=0$ part, and there is no term of $O(g^2T^2)$. Thus, from Eqs. (45), (46) and (42), we obtain the tachyonic mass terms

$$\int_0^\beta d\tau\int d^3x\frac{1}{2}\left(-\frac{\alpha_2}{24\pi^2}(g^2T)^2\right)\times\left(A_j^a(\mathbf{x})A_j^a(\mathbf{x})+\frac{3}{2}A_j^3(\mathbf{x})A_j^3(\mathbf{x})\right).\tag{47}$$

These "masses" are on the order of g^2T, which is the expected order of a magnetic mass. However, as in the four-dimensional Euclidean case, it is tachyonic.

VI. SUMMARY AND COMMENTS

In this paper, the $SU(2)$ gauge theory has been studied in the nonlinear gauge. To see the possibility of the ghost condensation, the auxiliary field $\varphi=\alpha_2\bar{B}$ with $\bar{B}=-B+igc\times\bar{c}$ has been introduced so as to preserve the BRS symmetry even if φ acquires a vacuum expectation value $\langle\varphi\rangle\neq 0$. We have shown $\langle\varphi\rangle\neq 0$ at the one-loop level in Euclidean space. The self-energy of gauge fields $\langle A_\mu^A A_\nu^B\rangle$ was calculated in the presence of the condensate $\langle\varphi\rangle\neq 0$. Whereas it behaves as $\propto(v^2/p^2)\ln(p^2/v)$ in the limit $p^2\to\infty$, it exhibits a tachyonic mass in the limit $p^2\to 0$. We also showed that a tachyonic mass is obtained in the maximal Abelian gauge, too. This result is different from that of Ref. [3], and coincides with the recent result of Ref. [18], if the Euclidean metric is employed. It is important to note that the values of the tachyonic masses in the present nonlinear gauge and in the maximal Abelian gauge are different from each other. That is, the tachyonic mass of gauge fields is gauge dependent.

Contrastingly, if we perform a straightforward calculation, the ghost condensation does not occur in Minkowski space. Because of the Minkowski metric, the square of a four momentum is $k^2=k_0^2-\mathbf{k}^2$. The different sign of the component g_{00} of the metric tensor plays a crucial role in carrying out momentum integrations. In some articles, the Minkowski metric is used, and the Wick rotation $k_0\to ik_0$ is assumed [4,18,19]. However, as was explained in Appendix D, this treatment is problematic. The appearance of the mass for off-diagonal gluons [4] is also dubious.

Although the ghost condensation does not occur in Minkowski space at zero temperature, it appears at high temperature. The point of this result is the reduction of a gauge theory in Minkowski space to that in three-dimensional Euclidean space. The condensation $\sqrt{g\langle\varphi\rangle}$ is of $O(g^2T)$, which is a typical mass scale in three-dimensional gauge theories. In the limit $\mathbf{p}^2\to 0$, the magnetostatic components of the gauge field acquire tachyonic masses of $O(g^2T)$.

The meaning of these tachyonic masses will be discussed in another paper.

Historically, the propagator which behaves as

$$\frac{k^2}{k^4+v^2}\tag{48}$$

was obtained by Gribov [20]. By restricting the functional integral of gauge fields inside the Gribov horizon, gluon propagators have this behavior [20,21]. Furthermore, the condition of this restriction is

$$\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2)^2+v^2}=c,\tag{49}$$

where $v^2=\kappa^4$ in the Gribov's notation [20], and $v^2=\gamma$ in the Zwanziger's notation [21]. It is emphasized that the model with Eq. (48) is not a physical Minkowski model but a Euclidean model [21]. To obtain physical results in Minkowski space from the Euclidean model, two-point functions of gauge invariant operators were discussed in Ref. [21]. Although we carried out a straightforward calculation in Sec. IV, a subtle analysis might be necessary to derive physical results in Minkowski space.

The condition (49) resembles the gap equation (13). Thus the ghost condensation might be related to the Gribov ambiguity [20]. This point will be discussed in the subsequent paper.

APPENDIX A

In this appendix, we make a comment on Eq. (6). As $\bar{B}=-B+ig\bar{c}\times c$, Eq. (5) is written as

$$\mathcal{L}_{gfgh}=B\cdot[\partial_\mu A_\mu+\alpha_2(ig\bar{c}\times c)-w]-\frac{\alpha_1+\alpha_2}{2}B^2+i\bar{c}\partial_\mu D_\mu c-\frac{\alpha_2}{2}(ig\bar{c}\times c)^2.$$

Usually, the auxiliary field $\phi\sim\alpha_2(ig\bar{c}\times c)$ is introduced [3,4,7]. In the present case, we obtain

$$B\cdot[\partial_\mu A_\mu+\alpha_2(ig\bar{c}\times c)-w]-\frac{\alpha_1+\alpha_2}{2}B^2+i\bar{c}\partial_\mu D_\mu c$$

$$-\phi\cdot(ig\bar{c}\times c)+\frac{1}{2\alpha_2}\phi^2.$$

However, in contrast with Eq. (6), the term which is proportional to B contains $\alpha_2(ig\bar{c}\times c)$ instead of ϕ. Therefore, when ϕ acquires a vacuum expectation value, we cannot see directly whether the BRS symmetry breaks or not. If we replace the auxiliary field as $\phi=\varphi+\alpha_2 B$, we can avoid this problem, and obtain Eq. (6).

The auxiliary field $\varphi^A(x)$ has another meaning. To see it, we rederive Eq. (6) by a functional integral method. In this method, a gauge-fixing and a ghost term come from

$$\Delta_f\delta(f(A)), \tag{A1}$$

where $f(A)=0$ is a gauge-fixing condition, and Δ_f is the corresponding Faddeev-Popov (FP) determinant. As $f(A)$, we choose

$$f^A(A)=\partial_\mu A_\mu^A-a^A+\varphi^A-w^A, \tag{A2}$$

where the function $a^A(x)$ and the constant w^A belong to the singlet representation of $SU(2)$, whereas $\varphi^A(x)$ belongs to the adjoint representation. Therefore the FP determinant becomes

$$\Delta_f=\det(\partial_\mu D_\mu+g\varphi\times)\propto\int D\bar{c}Dc\,e^{-i\bar{c}(\partial_\mu D_\mu+g\varphi\times)c}. \tag{A3}$$

Multiplying Eq. (A1) by the constants $\int Da\exp(-a^2/2\alpha_1)$, $\int DB\exp\{(\alpha_1/2)[B-(1/\alpha_1)(\partial_\mu A_\mu+\varphi-w)]^2\}$ and $\int D\varphi\exp(-\varphi^2/2\alpha_2)$, Eq. (A1) becomes

$$\int D\bar{c}DcD\varphi DB e^{[(\alpha_1/2)B^2-B(\partial_\mu A_\mu+\varphi-w)-i\bar{c}(\partial_\mu D_\mu+g\varphi\times)c-(\varphi^2/2\alpha_2)]},$$

where Eq. (A3) has been used. This expression gives the Lagrangian density (6). That is, $\varphi(x)$ is the function in the adjoint representation appearing in the gauge-fixing condition.

We note that the general case, which contains functions transforming nontrivially under $SU(N)$ in $f(A)$, is studied in Ref. [8].

APPENDIX B

It is necessary to calculate the integral

$$I_\mp(v)=\int\frac{d^Dk}{(2\pi)^D}(s_\mp)^{-a},\quad s_\mp(k,v)=U\mp k^2-iv, \tag{B1}$$

where U may contain external momenta and some parameters. As we consider not only the Euclidean case but also the Minkowski case, it is convenient to use the identity [11]

$$s_\mp^{-a}=\frac{i^a}{\Gamma(a)}\int_0^\infty(e^{-its_\mp})t^{a-1}dt,\quad \mathrm{Re}(a)>0,\mathrm{Im}(s_\mp)<0, \tag{B2}$$

which is derived from the integral representation of the gamma function $\Gamma(a)$. Substituting Eq. (B2) into Eq. (B1), we find

$$I_\mp(v)=\frac{i^a}{\Gamma(a)}\int_0^\infty dt\,t^{a-1}\int\frac{d^Dk}{(2\pi)^D}e^{-its_\mp}. \tag{B3}$$

Since

$$\int_{-\infty}^{\infty}dk_j e^{\pm itk_j^2}=\sqrt{\frac{\pm i\pi}{t}},$$

the k integral is performed as

$$\int d^Dk e^{\pm ik^2t}=\left(\frac{\pm i\pi}{t}\right)^{n/2}\left(\frac{\mp i\pi}{t}\right)^{(D-n)/2}$$

$$=(\pm i)^{n-(D/2)}\left(\frac{\pi}{t}\right)^{D/2}, \tag{B4}$$

where we have assumed $k^2=k_1^2+\cdots+k_n^2-k_{n+1}^2-\cdots-k_D^2$. Substituting Eq. (B4) into Eq. (B3), and using Eq. (B2) again, we obtain

$$I_\mp(v)=\frac{i^a}{\Gamma(a)}\frac{(\pm i)^n}{(\pm4\pi i)^{D/2}}\int_0^\infty dt e^{-i(U-iv)t}t^{a-D/2-1}$$

$$=\frac{(\pm i)^n}{(\pm4\pi)^{D/2}}\frac{\Gamma\left(a-\frac{D}{2}\right)}{\Gamma(a)}(U-iv)^{-(a-D/2)}. \tag{B5}$$

The factor $(\pm i)^n$ produces a difference between the Euclidean case and the Minkowski case. To see it, we set $U=0$, and consider the Euclidean case ($n=D$, in this paper). We find Eq. (B5) gives

$$\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(\mp k^2-iv)^a}=\frac{1}{(4\pi)^{D/2}}\frac{\Gamma\left(a-\frac{D}{2}\right)}{\Gamma(a)}$$

$$\times(-iv)^{-a}(\pm iv)^{D/2}. \tag{B6}$$

In the Minkowski case $(n=1)$, Eq. (B5) becomes

$$\int \frac{d^D k}{(2\pi)^D}\frac{1}{(\mp k^2 - iv)^a} = \pm i \frac{1}{(4\pi)^{D/2}}\frac{\Gamma\left(a-\frac{D}{2}\right)}{\Gamma(a)} \times(-iv)^{-a}(\mp iv)^{D/2}. \quad \text{(B7)}$$

The first $\pm$ sign on the right hand side of Eq. (B7), that is absent in Eq. (B6), plays a decisive role in Sec. IV. This $\pm$ sign is related to the direction of the so-called Wick rotation. The relation is explained in Appendix D.

APPENDIX C

To calculate Eq. (19), we consider the integral

$$L^{\pm}_{\mu\nu}(p) = \int \frac{d^4 k}{(2\pi)^4}\frac{k_\mu (k-p)_\nu}{(k^2 \pm iv)(k-p)^2}. \quad \text{(C1)}$$

Using the Feynmann parametrization and shifting the variable k_μ to $k_\mu + \chi p_\mu$, $L^{\pm}_{\mu\nu}(p)$ becomes

$$\int_0^1 d\chi \int \frac{d^4 k}{(2\pi)^4}\frac{k_\mu k_\nu + \chi(\chi-1)p_\mu p_\nu}{[k^2+\chi(1-\chi)p^2 \pm (1-\chi)iv]^2}. \quad \text{(C2)}$$

In four-dimensional Euclidean space, $k_\mu k_\nu$ in the numerator is replaced by $k^2\delta_{\mu\nu}/4$. We use the dimensional regularization, and apply Eq. (B5) with $n=D$ to Eq. (C2). The result is

$$L^{\pm}_{\mu\nu}(p) = -\frac{1}{(4\pi)^2}\frac{\delta_{\mu\nu}}{2}\int_0^1 d\chi[\chi(1-\chi)p^2 \pm (1-\chi)iv] \times[N(p^2,v,\chi)+1] + \frac{1}{(4\pi)^2}p_\mu p_\nu \times\int_0^1 d\chi \chi(\chi-1)N(p^2,v,\chi), \quad \text{(C3)}$$

where

$$N(p^2,v,\chi) = \frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln[\chi(1-\chi)p^2 \pm (1-\chi)iv].$$

Since $L^{\pm}_{\mu\nu}(p) = L^{\pm}_{\nu\mu}(p)$, we find

$$-\Pi^{11}_{\mu\nu}(p) = -g^2[L^{+}_{\mu\nu}(p)+L^{-}_{\mu\nu}(p)]$$
$$= -\frac{g^2}{(4\pi)^2}\Bigg[-\frac{1}{3}\left(p_\mu p_\nu + \frac{p^2}{2}\delta_{\mu\nu}\right)\times\left(\frac{1}{\varepsilon}-\gamma+\ln 4\pi\right) - \frac{p^2\delta_{\mu\nu}}{6} + \left(p_\mu p_\nu + \frac{p^2}{2}\delta_{\mu\nu}\right)\times\int_0^1 d\chi\chi(1-\chi)[\ln(\chi^2 p^4+v^2)+\ln(1-\chi)^2] + \frac{iv}{2}\delta_{\mu\nu}\int_0^1 d\chi(1-\chi)\ln\frac{\chi p^2+iv}{\chi p^2-iv}\Bigg].$$

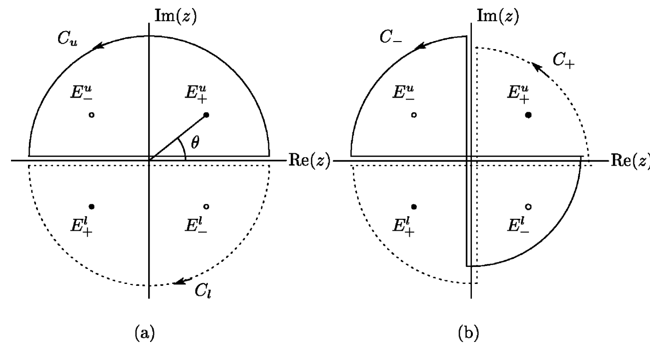

FIG. 2. Contours and poles in the complex k_0 plane.

Carrying out the χ integral, and using the relation

$$\ln\left(\frac{p^2-iv}{p^2+iv}\right) = -2i\tan^{-1}\frac{v}{p^2}, \quad \text{(C4)}$$

Eq. (20) is obtained.

APPENDIX D

In this appendix, we prove Eq. (30). Differentiating

$$K(v) = i\int_{-\infty}^{\infty} dk_0 \ln[(k_0^2-\mathbf{k}^2)^2+v^2]$$

by v, we find

$$\frac{dK(v)}{dv} = \int_{-\infty}^{\infty} dk_0\left(\frac{1}{k_0^2-E_+^2} - \frac{1}{k_0^2-E_-^2}\right), \quad \text{(D1)}$$

where $E_\pm^2 = \mathbf{k}^2 \pm iv$. We consider the complex integral

$$\int_C dz\left(\frac{1}{z^2-E_+^2} - \frac{1}{z^2-E_-^2}\right). \quad \text{(D2)}$$

If the anticlockwise contour C_u in Fig. 2(a) is chosen, Eq. (D2) gives

$$\frac{dK(v)}{dv} = 2\pi i[\mathrm{Res}(E^u_+) - \mathrm{Res}(E^u_-)] = \pi i\left(\frac{1}{E^u_+} - \frac{1}{E^u_-}\right), \quad \text{(D3)}$$

where $E^s_\pm (s=u,l)$ are the square roots of $E^2_\pm$:

$$E^u_\pm = \pm[(\mathbf{k}^2)^2+v^2]^{1/4}e^{\pm i\theta/2},$$
$$E^l_\pm = \mp[(\mathbf{k}^2)^2+v^2]^{1/4}e^{\pm i\theta/2},$$
$$\theta = \tan^{-1}(v/\mathbf{k}^2). \quad \text{(D4)}$$

Thus, integrating Eq. (D3) by v, we obtain

$$K(v) = 2\pi(E^u_+ + E^u_-).$$

Since

$$iV_{\rm gh}=\int \frac{d^3k}{(2\pi)^4}K(v),$$

Eq. (30) holds.

We note that we can choose the clockwise contour C_l. As $E^u_\pm=-E^l_\pm$, Eq. (D3) is obtained again.

We make a comment. To carry out k_0 integral in Minkowski space, the replacement $k_0\to ik_0$ is often used. However, we cannot apply it to integrals with $1/[(k_0^2-\mathbf{k}^2)^2+v^2]$. In Eq. (D1), we can apply it for the second part

$$\int_{-\infty}^{\infty}dk_0\frac{1}{k_0^2-E_-^2},$$

because the replacement $k_0\to ik_0$ corresponds to the contour C_+ in Fig. 2(b). Contrastingly, the first part

$$\int_{-\infty}^{\infty}dk_0\frac{1}{k_0^2-E_+^2}$$

contains the poles E^s_+ $(s=u,l)$ inside C_+. Therefore, if the replacement $k_0\to ik_0$ is applied to this part, the residues of E^s_+ $(s=u,l)$ must be added. Instead of taking these residues into account, we can use the contour C_-, which implies the replacement $k_0\to -ik_0$. The difference of the sign $\pm ik_0$ corresponds to the first $\pm$ sign in Eq. (B7).

APPENDIX E

We calculate $J_\pm$ defined in Eq. (36) for $D=3$. If we differentiate $J_\pm$ by v, we obtain

$$\frac{dJ_\pm(v)}{dv}=T\sum_n\int\frac{d^Dk}{(2\pi)^D}\frac{\pm i}{\mathbf{k}^2+\omega_n^2\pm iv} \tag{E1}$$

$$=\frac{\pm i}{\beta}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\mathbf{k}^2\pm iv}+\frac{\pm 2i}{\beta}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\mathbf{k}^2+\omega_n^2\pm iv}. \tag{E2}$$

Using Eq. (B5) with $a=1$ and $n=D$, the first term on the right hand side becomes

$$\frac{\pm i}{\beta}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\mathbf{k}^2\pm iv}=\frac{\mp i}{4\pi\beta}(\pm iv)^{1/2}, \tag{E3}$$

where we set $D=3$. In the same way, the second term becomes

$$\frac{\pm 2i}{\beta}\sum_{n=1}^{\infty}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\mathbf{k}^2+\omega_n^2\pm iv}=\frac{\pm 2i}{\beta}\frac{\Gamma(1-D/2)}{(4\pi)^{D/2}}\sum_{n=1}^{\infty}[\omega_n^2\pm iv]^{(D/2-1)}. \tag{E4}$$

We use the dimensional regularization, that is $D=3-2\varepsilon$, since Eq. (E4) diverges at $D=3$. Then, substituting Eq. (E3) and (E4) into Eq. (E2), and integrating it by v, we obtain

$$J_\pm(v)=-\frac{1}{6\pi\beta}(\pm iv)^{3/2}-\frac{2\pi^{3/2-\varepsilon}}{\beta^{4-2\varepsilon}}\Gamma\left(-\frac{3}{2}+\varepsilon\right)\times\sum_{n=1}^{\infty}\left[n^2\pm i\left(\frac{\beta}{2\pi}\right)^2v\right]^{3/2-\varepsilon}. \tag{E5}$$

Now we perform a high temperature expansion [16]. For $T\gg\sqrt{v}$, we find the expansion

$$\sum_{n=1}^{\infty}\left[n^2\pm i\left(\frac{\beta}{2\pi}\right)^2v\right]^{3/2-\varepsilon}=\zeta(-3+2\varepsilon)\pm i\left(\frac{3}{2}-\varepsilon\right)\frac{v\beta^2}{4\pi^2}\zeta(-1+2\varepsilon)-\frac{1}{8}(3-2\varepsilon)(1-2\varepsilon)\left(\frac{v\beta^2}{4\pi^2}\right)^2\zeta(1+2\varepsilon)+O(\beta^6), \tag{E6}$$

where $\zeta(\alpha)=\sum_{n=1}^{\infty}1/n^\alpha$ is the zeta function. Substituting Eq. (E6) into Eq. (E5), and using $\Gamma(\varepsilon-1/2)=-2\sqrt{\pi}[1-\varepsilon(2\ln 2+\gamma-2)]+O(\varepsilon^2)$ and $\zeta(1+2\varepsilon)=1/(2\varepsilon)+\gamma+O(\varepsilon)$, we obtain

$$J_\pm(v)=-\frac{1}{6\pi\beta}(\pm iv)^{3/2}-\frac{\pi^2}{45\beta^4}\pm\frac{iv}{12\beta^2}+\frac{v^2}{32\pi^2}\left(\frac{1}{\varepsilon}+\gamma-\ln 4\pi-\ln T^2\right)+O(\beta^2v^3). \tag{E7}$$

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